of Pringsheim, who has greatly extended the class of functions to which Fourier's Integral Theorem applies, has been used.

"Two appendices are added. The first deals with 'Practical Harmonic Analysis and Periodogram Analysis.' In the second a bibliography of the subject is given.

"The functions treated in this book are 'ordinary' functions. An interval (a, b) for which $f(x)$ is defined can be broken up into a finite number of open partial intervals, in each of which the function is monotonic. If infinities occur in the range, they are isolated and finite in number. Such functions will satisfy most of the demands of the Applied Mathematician.

"The modern theory of integration, associated chiefly with the name of Lebesgue, has introduced into the Theory of Fourier's Series and Integrals functions of a far more complicated nature. Various writers, notably W. H. Young, are engaged in building up a theory of these and allied series much more advanced than anything treated in this book. These developments are in the meantime chiefly interesting to the Pure Mathematician specialising in the Theory of Functions of a Real Variable. My purpose has been to remove some of the difficulties of the Applied Mathematician."

Contents—Historical introduction, 1–15; Chapter I: Rational and irrational numbers, 16–28; II: Infinite sequences and series, 29–48; III: Functions of a single variable. Limits and continuity, 49–75; IV: The definite integral, 76–121; V: The theory of infinite series whose terms are functions of a single variable, 122–168; VI: Definite integrals containing an arbitrary parameter, 169–195; VII: Fourier's series, 196–247; VIII: The nature of the convergence of Fourier's series, 248–263; IX: The approximation curves and Gibbs's phenomenon in Fourier's series, 264–282; X: Fourier's integrals, 283–294; Appendix I: Practical harmonic analysis and periodogram analysis, 295–301; II: Bibliography, 302–317; List of authors quoted, 318–319; General index, 320–323.

Each chapter concludes with bibliographical "References." The names of the following Americans occur in the work: BÔCHER, BYERLY, FORD, GRONWALL, JACKSON, C. N. MOORE, and VAN VLECK.

Plane and Solid Analytic Geometry. By W. F. OSGOOD and W. C. GRAUSTEIN. New York, The Macmillan Company, 1921. 12mo. 17 + 614 pp. Price $3.75.

Preface: "The object of an elementary college course in Analytic Geometry is twofold: it is to acquaint the student with new and interesting and important geometrical material, and to provide him with powerful tools for the study, not only of geometry and pure mathematics, but in no less measure of physics in the broadest sense of the term, including engineering.

"To attain this object, the geometrical material should be presented in the simplest and most concrete form, with emphasis on the geometrical content, and illustrated, whenever possible, by its relation to physics. This principle has been observed throughout the book. Thus, in treating the ellipse, the methods actually used in the drafting room for drawing an ellipse from the data commonly met in descriptive geometry are given a leading place. The theorem that the tangent makes equal angles with the focal radii is proved mechanically: a rope which passes through a pulley has its ends tied at the foci and is drawn taut by a line fastened to the pulley. Moreover, the meaning of foci in optics and acoustics is clearly set forth. Again, there is a chapter on the deformations of an elastic plane under stress, with indications as to the three-dimensional case (pure strain, etc.).

"The methods of analytic geometry, even in their simplest forms, make severe demands on the student's ability to comprehend the reasoning of higher mathematics. Consequently, in presenting them for the first time, purely algebraic difficulties, such as are caused by literal coefficients and long formal computations, should be avoided. The authors have followed this principle consistently, beginning each new subject of the early chapters with the discussion of a simple special, but typical, case, and giving immediately at the close of the paragraph simple examples of the same sort. They have not, however, stopped here, but through carefully graded problems, both of geometric and of analytic character, have led the student to the more difficult applications of the methods, and collections of examples at the close of the chapters contain such as put to the test the initiative and originality of the best students.

"As a result of this plan the presentation is extraordinarily elastic. It is possible to make the treatment of any given topic brief without rendering the treatment of later topics unintelligible, and thus the instructor can work out a course of any desired extent. For example, one freshman

course at Harvard devotes about thirty periods to analytic geometry and the material consists of the essential parts of the first nine chapters. Another freshman course gives twice the time to analytic geometry (the students having already had trigonometry), taking up determinants and the descriptive properties of the quadric surfaces, and also devoting more time to the less elementary applications of the methods of analytic geometry. The advanced courses in the calculus and mechanics require the material of the later chapters. In fact, a thorough elementary treatment of the rudiments of Solid Analytic Geometry is indispensable for the understanding of standard texts on applied mathematics. It is true that these texts are chiefly Continental. But we shall never have American treatises which are up to the best scientific standards of the day until the subjects above mentioned are available in simply intelligible form for the undergraduate.

"The subject of loci is brought in early through a brief introductory chapter, and problems in loci are spread throughout the book. A later chapter is devoted to a careful explanation of the method of auxiliary variables. There is a chapter on determinants, with applications both to analytic geometry and to linear equations. Diameters and poles and polars in the plane and in space receive a thorough treatment. Cylindrical and spherical coördinates and quadric surfaces are illumined by the concept of triply orthogonal systems of surfaces. The reduction of the general equation of the second degree in space to normal forms by translations and rotations is sketched and illustrated by numerical examples.

"The question may be asked: In so extensive a treatment of analytic geometry should not, for example, homogeneous coördinates find a place? The authors believe that the student, before proceeding to the elaborate methods of modern geometry, should have a thorough knowledge both of the material and the methods which may fairly be called elementary, and they felt that a book which, avoiding the conciseness of some of the current texts and the looseness of others, is clear because it is rigorous will meet a real need.

"This book is designed to be at once an introduction to the subject and a handbook of the elements. May it serve alike the needs of the future specialist in geometry, the analyst, the mathematical physicist, and the engineer."

Contents. *Plane analytic geometry*—Introduction: Directed line-segments. Projections, 1–6; Chapter I: Coördinates. Curves and equations, 7–26; II: The straight line, 27–52; III: Applications, 53–64; IV: The circle, 65–78; V: Introductory problems in loci. Symmetry of curves, 79–87; VI: The parabola, 88–100; VII: The ellipse, 101–123; VIII: The hyperbola, 124–153; IX: Certain general methods, 154–192; X: Polar coördinates, 193–215; XI: Transformation of coördinates, 216–234; XII: The general equation of the second degree, 235–260; XIII: A second chapter on loci. Auxiliary variables. Inequalities, 261–287; XIV: Diameters. Poles and polars, 288–329; XV: Transformations of the plane. Strain, 330–359; XVI: Determinants and their applications, 360–404. *Solid analytic geometry*—Chapter XVII: Projections. Coördinates, 405–420; XVIII: Direction cosines. Direction components, 420–443; XIX: The plane, 444–469; XX: The straight line, 470–497; XXI: The plane and the straight line. Advanced methods, 498–522; XXII: Spheres, cylinders, cones. Surfaces of revolution, 523–547; XXIII: Quadric surfaces, 548–583; XXIV: Spherical and cylindrical coördinates. Transformation of coördinates, 584–606; Index, 607–614.

Einstein's Theories of Relativity and Gravitation. A Selection of Material from the Essays submitted in the Competition for the Eugene Higgins Prize of $5,000. Compiled and edited, and introductory matter supplied, by J. M. BIRD, associate editor of the *Scientific American*. New York, Scientific American Publishing Co., 1921. 12mo. 14 + 345 pp. Price $2.00.

We have already referred [*1921*, 191–192; 269] to the prize offered by the *Scientific American*, to the conditions of its award, and to essays presented in competition for it. In the volume under review the prize essay is not given till pages 169–180 and this is followed by a dozen other essays. It was designed by the editor that "Each essay should be made easier of reading by the examination of those preceding it; at the same time each, by the choice of ground covered and by the emphasis on points not brought out sharply by its predecessors, should throw new light upon these predecessors." Chapters II–VI, pages 19–